Online Appendix—Railway Restructuring and Organizational Choice: Network Quality and Welfare Impacts

David Besanko and Shana Cui

1 Cost asymmetry model

In this Online Appendix we explore the impact of cost asymmetry on network quality, consumer surplus and social welfare under HS and VS. We assume that the *average* marginal operating cost is fixed at γ_k, where $k \in \{V, H\}$, and the marginal operating cost for each firm γ_{ik} is given by:

$$\begin{aligned} \gamma_{1k} &= \alpha\gamma_k \\ \gamma_{2k} &= (2-\alpha)\gamma_k, \\ \alpha &\in [0,1], k \in \{V, H\}. \end{aligned}$$

1.1 Equilibrium quality and price conditions with cost asymmetry

Using standard analysis, the Bertrand-Nash equilibrium transport prices under each structure can be shown to be

$$\begin{aligned} P_{1V}^*(\mathbf{q}_V) &= \frac{(1-b_V)(a+q_V)+(c+\alpha\gamma_V)}{2-b_V} + \frac{2b_V(1-\alpha)\gamma_V}{(2-b_V)(2+b_V)}. \\ P_{2V}^*(\mathbf{q}_V) &= \frac{(1-b_V)(a+q_V)+[c+(2-\alpha)\gamma_V]}{2-b_V} - \frac{2b_V(1-\alpha)\gamma_V}{(2-b_V)(2+b_V)}. \end{aligned}$$

$$\begin{aligned} P_{1H}^*(\mathbf{q}_H) &= \frac{(1-b_H)(a+q_{1H})+\alpha\gamma_H+\eta_H}{2-b_H} + \frac{b_H[q_{1H}-q_{2H}]}{(2-b_H)(2+b_H)} + \frac{2b_H(1-\alpha)\gamma_H}{(2-b_H)(2+b_H)}. \\ P_{2H}^*(\mathbf{q}_H) &= \frac{(1-b_H)(a+q_{2H})+(2-\alpha)\gamma_H+\eta_H}{2-b_H} + \frac{b_H[q_{2H}-q_{1H}]}{(2-b_H)(2+b_H)} - \frac{2b_H(1-\alpha)\gamma_H}{(2-b_H)(2+b_H)}. \end{aligned}$$

The induced average equilibrium quantity under each structure with cost asymmetry can be rewritten as

$$\overline{X}_V^* = \frac{X_{1V}^* + X_{2V}^*}{2} = \frac{1}{2-b_V}(a + q_V^* - c_V^* - \gamma_V) = X_{V\_symmetry}^*.$$

$$\begin{aligned}\overline{X}_H^* &= \frac{X_{1H}^* + X_{2H}^*}{2} = \frac{1}{2(2-b_H)}[2(a-\eta_H-\gamma_H) + (q_{1H}^* + q_{2H}^*)] \\ &= \frac{1}{2(2-b_H)}[2(a-\eta_H-\gamma_H) + 2q_H^*] = \frac{a + q_H^* - \eta_H - \gamma_H}{(2-b_H)} \\ &= X_{H\_symmetry}^*,\end{aligned}$$

where the overbar denotes average and the subscript "$\_symmetry$" refers to the case examined in the paper in which the marginal transport costs of the two firms are the same, or equivalently, the case of $\alpha = 1$. We see that cost asymmetry leaves the average quantity the same as it was under cost symmetry.

Under HS, the first-order condition for network quality with cost asymmetry is given by

$$(P_{iH}^* - \gamma_H - \eta_H)\left[\frac{\partial X_{iH}^D}{\partial P_i}\frac{\partial P_{iH}^*}{\partial q_{iH}} + \frac{\partial X_{iH}^D}{\partial P_j}\frac{\partial P_{jH}^*}{\partial q_{iH}} + \frac{\partial X_{iH}^D}{\partial q_{iH}}\right] + \frac{\partial P_{iH}^*(\mathbf{q}_H)}{\partial q_{iH}} X_{iH}^D(P_{1H}^*(\mathbf{q}_H), P_{2H}^*(\mathbf{q}_H), \mathbf{q}_H)) - \lambda q_{iH} = 0,$$
$$i \in \{1,2\}, j \in \{1,2\}, i \neq j.$$

Thus, the Nash equilibrium network qualities under HS with cost asymmetry are given by

$$\begin{cases} q_{1H}^* = q_{H\_symmetry}^* + \frac{\phi(b_H)b_H(2-b_H)(1-\alpha)}{2\lambda(1-b_H)(2+b_H) - 2\phi(b_H)(1+b_H)(2-b_H)}\gamma_H. \\ q_{2H}^* = q_{H\_symmetry}^* - \frac{\phi(b_H)b_H(2-b_H)(1-\alpha)}{2\lambda(1-b_H)(2+b_H) - 2\phi(b_H)(1+b_H)(2-b_H)}\gamma_H. \end{cases}$$

where

$$q_{H\_symmetry}^* = \frac{\phi(b_H)}{\lambda - \phi(b_H)}(a - \eta_H - \gamma_H).$$

It immediately follows that the average equilibrium network quality under HS with cost asymmetry is equal to the equilibrium network quality under HS with cost symmetry:

$$\overline{q}_H^* = \frac{q_{1H}^* + q_{2H}^*}{2} = q_{H\_symmetry}^*.$$

Also, it can be seen that the average equilibrium transport price under each structure with cost asymmetry is equal to the equilibrium transport price under each structure with cost symmetry:

$$\begin{aligned}\overline{P}_H^* &= \frac{P_{1H}^* + P_{2H}^*}{2} = P_{H\_symmetry}^*. \\ \overline{P}_V^* &= \frac{P_{1V}^* + P_{2V}^*}{2} = P_{V\_symmetry}^*.\end{aligned}$$

Under VS, the first-order condition for network quality with cost asymmetry is given by

$$(c-\eta_V)\left\{\begin{array}{c} \left[\frac{\partial X_{1V}^D}{\partial P_1}\frac{\partial P_{1V}^*}{\partial q_V}+\frac{\partial X_{1V}^D}{\partial P_2}\frac{\partial P_{2V}^*}{\partial q_V}+\frac{\partial X_{1V}^D}{\partial q_V}\right] \\ +\left[\frac{\partial X_{2V}^D}{\partial P_1}\frac{\partial P_{1V}^*}{\partial q_V}+\frac{\partial X_{2V}^D}{\partial P_2}\frac{\partial P_{2V}^*}{\partial q_V}+\frac{\partial X_{2V}^D}{\partial q_V}\right]\end{array}\right\}-\lambda q_V=0.$$

Because the term $\left\{\begin{array}{c} \left[\frac{\partial X_{1V}^D}{\partial P_1}\frac{\partial P_{1V}^*}{\partial q_V}+\frac{\partial X_{1V}^D}{\partial P_2}\frac{\partial P_{2V}^*}{\partial q_V}+\frac{\partial X_{1V}^D}{\partial q_V}\right] \\ +\left[\frac{\partial X_{2V}^D}{\partial P_1}\frac{\partial P_{1V}^*}{\partial q_V}+\frac{\partial X_{2V}^D}{\partial P_2}\frac{\partial P_{2V}^*}{\partial q_V}+\frac{\partial X_{2V}^D}{\partial q_V}\right]\end{array}\right\}$ with asymmetric firms is equal to that with symmetric firms, the first-order condition remain the same with asymmetric firms. Thus, the equilibrium quality with asymmetric firms under VS is equal to the equilibrium quality with symmetric firms under VS, which means that

$$q_V^*(c)=q_V^*(c)_{\_symmetry}=\left\{\begin{array}{cc} 0 & \text{if } c\leq\eta_V \\ \frac{2(c-\eta_V)}{\lambda(2-b_V)} & \text{if } c\in(\eta_V,\overline{c}) \\ \overline{q} & \text{if } c\geq\overline{c}\end{array}\right. .$$

The first-order condition for an interior solution of access charge with cost asymmetry is

$$\begin{aligned}\frac{dSW(c)}{dc} &= \sum_{i=1}^{2}\left[\frac{\partial U_V(\mathbf{X}_{1V}^*(c),\mathbf{X}_{2V}^*(c),\mathbf{q}_V^*(c))}{\partial X_i}-(\gamma_{iV}+\eta_V)\right]\frac{dX_{iV}^*(c)}{dc} \\ &\quad +\sum_{i=1}^{2}\left[\frac{\partial U_V(\mathbf{X}_V,\mathbf{q}_V)}{\partial q_i}\right]\frac{dq_V^*(c)}{dc}-\lambda q_V^*(c)\frac{dq_V^*(c)}{dc} \\ &= 0.\end{aligned}$$

It can be seen that

$$\begin{aligned}&\sum_{i=1}^{2}\left[\frac{\partial U_V(\mathbf{X}_{1V}^*(c),\mathbf{X}_{2V}^*(c),\mathbf{q}_V^*(c))}{\partial X_i}-(\gamma_{iV}+\eta_V)\right]\frac{dX_{iV}^*(c)}{dc} \\ =\ &[\frac{\partial U_V(\mathbf{X}_{1V}^*(c),\mathbf{X}_{2V}^*(c),\mathbf{q}_V^*(c))}{\partial X_1}+\frac{\partial U_V(\mathbf{X}_{1V}^*(c),\mathbf{X}_{2V}^*(c),\mathbf{q}_V^*(c))}{\partial X_2}-2(\gamma_V+\eta_V)](-\frac{1}{(2-b_V)}) \\ =\ &\sum_{i=1}^{2}\left[\frac{\partial U_V(\mathbf{X}_{1V}^*(c),\mathbf{X}_{2V}^*(c),\mathbf{q}_V^*(c))}{\partial X_i}-(\gamma_V+\eta_V)\right]\frac{dX_{iV}^*(c)}{dc}.\end{aligned}$$

In the first part of the first-order condition, the terms $\frac{\partial U_V(\mathbf{X}_{1V}^*(c),\mathbf{X}_{2V}^*(c),\mathbf{q}_V^*(c))}{\partial X_i}$ and $\frac{dX_{iV}^*(c)}{dc}$ do not change whether we have symmetric or asymmetric firms, and the second part of the first-order condition $\sum_{i=1}^{2}\left[\frac{\partial U_V(\mathbf{X}_V,\mathbf{q}_V)}{\partial q_i}\right]\frac{dq_V^*(c)}{dc}-\lambda q_V^*(c)\frac{dq_V^*(c)}{dc}$ also does not change with or without symmetric firms. Thus, the first-order condition for access charge under VS remain the same with or without symmetric firms. This means that the access charge with cost asymmetry is equal to that with cost symmetry, which is given by

$$c^* = c^*_{\_symmetry}$$

1.2 Consumer surplus with cost asymmetry

Consumer surplus (expressed as a function of quantity) is given by

$$\begin{aligned} CS_k^*(\mathbf{X}_k) &= U_k(\mathbf{X}_k) - \sum_{i=1}^{2} P_{ik}(\mathbf{X}_k)X_{ik} \\ &= \sum_{i=1}^{2}(a+q_{ik})X_{ik} - \frac{1}{2(1+b_k)}(X_{1k}^2 + 2b_kX_{1k}X_{2k} + X_{2k}^2) - \sum_{i=1}^{2} P_{ik}X_{ik}, \\ i &\in \{1,2\}, j \in \{1,2\}, i \neq j, k \in \{V, H\}. \end{aligned}$$

where

$$P_{ik}(\mathbf{X}_k) = \frac{\partial U_k(\mathbf{X}_k)}{\partial X_{ik}} = a + q_{ik} - \frac{1}{(1+b_k)}X_{ik} - \frac{b_k}{(1+b_k)}X_{jk}, i \in \{1,2\}, j \in \{1,2\}, i \neq j.$$

Thus,

$$CS_k^*(\mathbf{X}_k) = \frac{1}{2(1+b_k)}(X_{1k}+X_{2k})^2 - \frac{(1-b_k)}{(1+b_k)}X_{1k}X_{2k}, k \in \{V, H\}.$$

Note that

$$\begin{aligned} \frac{\partial CS_k^*(\mathbf{X}_k)}{\partial X_{1k}} &= \frac{X_{1k} + b_kX_{2k}}{1+b_k} > 0. \\ \frac{\partial CS_k^*(\mathbf{X}_k)}{\partial X_{2k}} &= \frac{X_{2k} + b_kX_{1k}}{1+b_k} > 0. \end{aligned}$$

Thus, the Hessian is given by

$$\begin{bmatrix} \frac{\partial^2 CS_k^*(\mathbf{X}_k)}{\partial X_{1k}^2} & \frac{\partial^2 CS_k^*(\mathbf{X}_k)}{\partial X_{1k}\partial X_{2k}} \\ \frac{\partial^2 CS_k^*(\mathbf{X}_k)}{\partial X_{1k}\partial X_{2k}} & \frac{\partial^2 CS_k^*(\mathbf{X}_k)}{\partial X_{2k}^2} \end{bmatrix} = \frac{1}{1+b_k}\begin{bmatrix} 1 & b_k \\ b_k & 1 \end{bmatrix}.$$

The Hessian is positive semi-definite, which implies that $CS_k^*(\mathbf{X}_k)$ is a strictly increasing, strictly convex function. Thus, for any two quantity vectors $\mathbf{X}_k^A$ and $\mathbf{X}_k^B$ and any scalar $t \in (0,1)$, it follows that

$$CS_k^*(t\mathbf{X}_k^A + (1-t)\mathbf{X}_k^B) < tCS_k^*(\mathbf{X}_k^A) + (1-t)CS_k^*(\mathbf{X}_k^B). \tag{1}$$

In particular, let $t = \frac{1}{2}$ and

$$\begin{aligned} \mathbf{X}_k^A &= (X_{1k}^*, X_{2k}^*) \\ \mathbf{X}_k^B &= (X_{2k}^*, X_{1k}^*) \,. \end{aligned}$$

Note that,

$$\begin{aligned} CS_k^* \left(\frac{1}{2}\mathbf{X}_k^A + \frac{1}{2}\mathbf{X}_k^B \right) &= CS_k^* \left(\frac{1}{2}\left(X_{1k}^*, X_{2k}^*\right) + \frac{1}{2}\left(X_{2k}^*, X_{1k}^*\right) \right) \\ &= CS_k^* \left(\frac{X_{1k}^* + X_{2k}^*}{2}, \frac{X_{1k}^* + X_{2k}^*}{2} \right) \\ &= CS_k^*(X_{k\_symmetry}^*, X_{k\_symmetry}^*) \\ &= CS_{k\_symmetry}^*, \qquad (2) \end{aligned}$$

where the third equality follows because we have seen that $X_{k\_symmetry}^* = \frac{1}{2}\left(X_{1k}^* + X_{2k}^*\right)$ for $k \in \{V, H\}$. Given (1), (2) implies

$$\begin{aligned} CS_{k\_symmetry}^* &< \frac{1}{2}CS_k^*(X_{1k}^*, X_{2k}^*) + \frac{1}{2}CS_k^*(X_{2k}^*, X_{1k}^*) \qquad (3) \\ &= CS_k^*(X_{1k}^*, X_{2k}^*). \qquad (4) \end{aligned}$$

The equality follows because

$$\begin{aligned} CS_k^*(X_{1k}^*, X_{2k}^*) &= \frac{1}{2(1+b_k)}(X_{1k}^* + X_{2k}^*)^2 - \frac{(1-b_k)}{(1+b_k)}X_{1k}^* X_{2k}^* \\ &= \frac{1}{2(1+b_k)}(X_{2k}^* + X_{1k}^*)^2 - \frac{(1-b_k)}{(1+b_k)}X_{2k}^* X_{1k}^* \\ &= CS_k^*(X_{2k}^*, X_{1k}^*). \end{aligned}$$

Thus, under either organizational structure, consumer surplus with cost asymmetry exceeds consumer surplus with cost symmetry.

1.3 Social welfare with cost asymmetry

We can show that under VS

$$SW_V^* = SW_{V\_Symmetry}^* - \frac{(1-\alpha)^2(1+b_V)}{(1-b_V)(2+b_V)^2}\gamma_V^2,$$

where

$$SW_{V\_Symmetry}^* = (3 - 2b_V)(X_V^*)^2 + 2(c_V^* - \eta_V)X_V^* - \frac{\lambda}{2}\left(q_V^*\right)^2.$$

Thus

$$SW_V^* < SW_{V\_Symmetry}^*,$$

i.e., under VS, social welfare with symmetric firms is less than it is with asymmetric firms.

Under HS, we can show that

$$
\begin{aligned}
&SW_H^* = SW_{H\_Symmetry}^* \\
&+\frac{2(1-\alpha)(1+b_H)b_H\gamma_H}{(1-b_H)}\left\{[\frac{\phi(b_H)}{(2-b_H^2)}-\frac{4}{(2+b_H)(2-b_H)^2}]\frac{\lambda(a-\eta_H-\gamma_H)}{[\lambda-\phi(b_H)]}+\frac{2a}{(2-b_H)(2+b_H)}\right\} \\
&-\frac{(1+b_H)}{(1-b_H)}\left[\frac{(5-4\alpha)b_H+2}{(2+b_H)(2-b_H)^2}\gamma_H^2+\xi_1\xi_2\right] \\
&+\frac{(1-\alpha)^2(2-b_H^2)b_H\gamma_H^2\left[\begin{array}{c} 2\lambda(1-b_H^2)(2-b_H)(2+b_H) \\ -2(1+b_H)(2-b_H^2) \\ -\lambda b_H(1-b_H)(2-b_H^2) \end{array}\right]}{(1-b_H)[\lambda(1-b_H)(2-b_H)(2+b_H)^2-2(1+b_H)(2-b_H^2)]^2},
\end{aligned} \tag{5}
$$

where

$$SW_{H\_Symmetry}^* = (3-2b_H)(X_H^*)^2 - \lambda\left(q_H^*\right)^2.$$

As a check, note that when $\alpha = 1$, $SW_H^* = SW_{H\_Symmetry}^*$. Given the expression in (5), we have been unable to sign $SW_H^* - SW_{H\_Symmetry}^*$ unambiguously.

1.4 Computational analysis

We now use computational analysis to show how the cost asymmetry would affect the equilibrium quality, consumer surplus and social welfare under two structures. Recall that we assume that with cost asymmetry, the marginal operating cost for each firm γ_{ik} is a fraction of the average marginal operating cost γ_k:

$$
\begin{aligned}
\gamma_{ik} &= \alpha\gamma_k \\
\gamma_{jk} &= (2-\alpha)\gamma_k, \\
\alpha &\in [0,1], k\in\{V,H\}, i\in\{1,2\}, j\in\{1,2\}, i\neq j
\end{aligned}
$$

Thus, in our model, we use $\alpha \in [0,1]$ to represent the degree of cost asymmetry (where $\alpha = 1$ refers to cost symmetry, the case analyzed in the paper). We maintain our assumption that marginal network costs are the same across each structure, i.e., $\gamma_V = \gamma_H = \gamma$ and $\eta_V = \eta_H = \eta$. When we explore the effect of α on the probability of HS dominates VS in terms of q, CS, SW or all, we let the other six parameters γ, η, a, b_V, b_H, λ vary in their designated ranges, which is showed in Table 1.

Parameter	λ	a	η	γ	b_V	b_H
Variation range	$[1,10]$	$[6,9]$	$[0,3]$	$[0,3]$	$[0,0.99]$	$[-0.99,0.99]$

Table 1: Variation range of parameter

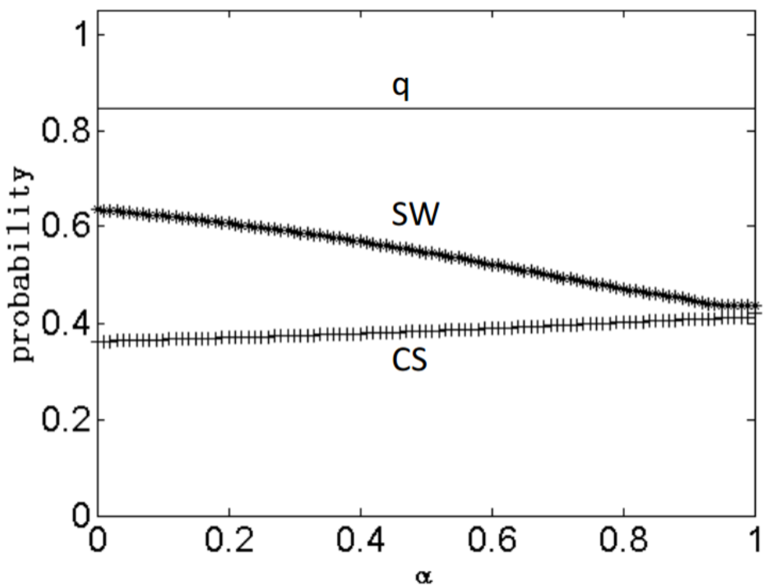

Figure 1: The probability of higher q, CS, SW under HS, respectively

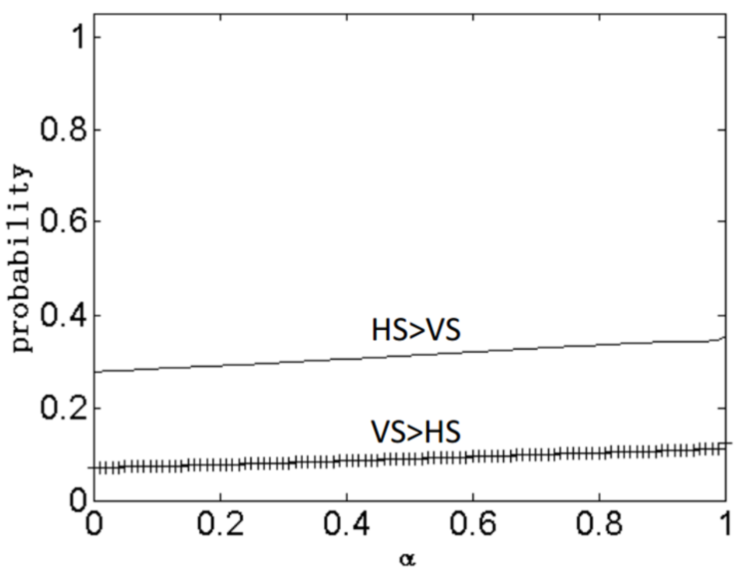

Figure 2: The probability that HS strongly dominates VS and VS strongly dominates HS

Figure 1 shows three lines, each indicating how α affects the proportion of parameterizations for which $\overline{q}_H^* > q_V^*$, $CS_H^* > CS_V^*$, and $SW_H^* > SW_V^*$ when the other parameters vary in their designated ranges.

First, consistent with our analytical result above, we see that the proportion of equilibria for which $\overline{q}_H^* > q_V^*$ is not affected by α, i.e., the ranking of (average) network quality under the two structures is unaffected by the presence of cost asymmetry. Second, the proportion of equilibria for which $CS_H^* > CS_V^*$ is virtually unchanged when we allow for cost asymmetry. Third, the proportion of equilibria in which $SW_H^* > SW_V^*$ slowly increases as firms' transport costs become more asymmetric (lower α). Thus, cost asymmetry is a factor that tend to favor HS when it comes to the ranking of social welfare across the two structures.

Figure 2 (which is analogous to Figure 4 in the paper) shows that the probability that HS strongly dominates VS and the probability that VS strongly dominates HS both slightly decrease when cost asymmetry is considered. Thus, while cost asymmetry slightly "muddies the water" when it comes to cases in which one structure strictly dominates the other on all metrics, Figure 2 is consistent with the implication of Figure 4 in the paper that there is a higher proportion of parameterizations for which HS dominant than there is for VS.[1]

Summing up, cost asymmetry does not change the comparison of equilibrium qualities under two structures, and it only slightly changes the proportion of equilibria in which $CS_H^* > CS_V^*$ and $SW_H^* > SW_V^*$. Cost asymmetry also has a limited effect on the probability that one structure strongly dominates the other one.

[1] We have also created a version of Figure 3 in the paper in which we allow α to vary between 0 and 1 along with variations in the other parameters. A comparison of this "cost asymmetry" version of Figure 3 with the version in the paper reveals that cost asymmetry has only a very slight effect on the proportion of cases for for which $\overline{q}_H^* > q_V^*$, $CS_H^* > CS_V^*$, and $SW_H^* > SW_V^*$ as we vary λ, b_V,b_h, a, η, and γ. A copy of this figure is available from the authors on request.